Towards improving the Database of Local Fields

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Two-paragraph summary

The p-adic field section of the LMFDB tabulates degree n extensions of $\mathbb{Q}_p$, including for all $n \leq 15$ and $p \leq 199$. For example, always up to isomorphism, there are 795 nonic extensions $K/\mathbb{Q}_3$ and 1823 octic extensions $K/\mathbb{Q}_2$. Interesting invariants include *visible slopes*, *hidden slopes*, and *Galois groups*.

The main framework for improvement is to **focus first on visible slopes**. Here there is a strong general theory valid for general K/F, not just the case $F = \mathbb{Q}_p$. It centers on **Krasner-Monge near-canonical polynomials** for totally ramified extensions K/F. These polynomials let one collect all extensions of a given F with given visible slopes into a single parameterized family, and the dependence on F is mild. The family structure then facilitates the investigation of hidden slopes and Galois groups.

Overview

1. **Introduction**, including a tour of the database.

2. **The ramification invariant** I of an extension K/F, as captured in the totally wild degree p^w case by

$$\begin{array}{ll} \textit{heights} & \langle h_1, \ldots, h_w \rangle, \\ \textit{slopes} & [s_1, \ldots, s_w], \\ \text{or } \textit{rams} & (r_1, \ldots, r_w). \end{array}$$

3. **The set $\mathcal{I}$ of possible ramification invariants.**

4. **From ramification invariants to pictures.**

5. **From pictures to near-canonical polynomials.**

6. **Hidden slopes and Galois groups** in two sample families.

1.1. Notation for classifyng extensions

Let $n \in \mathbb{Z}_{\geq 1}$ and let F be a field. An important problem is to describe the set $F(n)$ of isomorphism classes of separable field extensions K/F of degree n.

Let G run over conjugacy classes of transitive subgroups of S_n. Then Galois theory gives a natural decomposition

$$F(n) = \coprod_G F(G).$$

One would like to describe each $F(G)$ individually.

Now let F be a p-adic field, i.e. a finite extension of $\mathbb{Q}_p$ or $\mathbb{F}_p((t))$, with uniformizer $\pi \in \Pi \subset \mathcal{O} \subset F$ as usual. Then every K/F has a discriminant ideal Π^c, giving

$$F(n) = \coprod_G \coprod_c F(G, c).$$

The sets $F(G, c)$ are finite and one would like to describe them individually.

1.2. Overview of the 795 nonic 3-adic fields

There are 81 nonzero $|\mathbb{Q}_3(G, c)|$ with 22 Galois groups G and 16 discriminant exponents c involved. On the table, groups G are sorted first by the number of cubic subfields: ≥ 2, 1, and then 0. In the third column, $A = \mathrm{Aut}(K/\mathbb{Q}_3)$ is the centralizer of the Galois group G.

$\|G\|$	G	$\|A\|$	0	9	10	12	13	15	16	18	19	20	21	22	23	24	25	26
9	$9T2$	9				*1*												
18	$9T4$	3		*2*		*1*		6, *3*	3		9							
18	$9T5$									1								
36	$9T8$					1		2		3	3							
9	$9T1$	9	**1**			*2*								9				
18	$9T3$									1				1				3
27	$9T6$	3				*2*								6				
27	$9T7$	3				*1*			3									
54	$9T10$								6	11				8				24
54	$9T11$					2			1	8				9				
54	$9T12$	3									9		27					
54	$9T13$			*2*		*1*		*2*		3	3		9					
81	$9T17$	3				*9*				9				18				
108	$9T18$						2	4		3	12		18	9				
162	$9T20$	3					6	12		9	45			27			81	
162	$9T21$											27				27		54
162	$9T22$			*6*		*3*		*6*					9	9	27			
324	$9T24$								6	12	9	27	9		27	27	27	
36	$9T9$					1			1									
72	$9T14$				1					3								
72	$9T16$				1					3								
144	$9T19$			2		2	2	6	2		6							

Bold=Unramified *Italic=partially ramified* Regular=totally ramified

1.3. Tour of the p-adic section of the LMFDB

As said earlier, the LMFDB currently contains the sets $\mathbb{Q}_p(n)$ for all $p \leq 199$ and all $n \leq 15$, with information on each field K. E.g., the field K labeled 3.9.21.20 is defined by $x^9 + 12x^6 + 18x^4 + 3$.

The degree n of any field factors as utp^w with u and t its unramified and tamely ramified parts. There are w wild slopes $\hat{s}_1 \leq \cdots \leq \hat{s}_w$, as introduced in the next section. The "slope content" of our example (not directly given in the LMFDB) is $[\hat{s}_1, \hat{s}_2]_t^u = [2, \frac{17}{6}]_1^1 = [2, \frac{17}{6}]$.

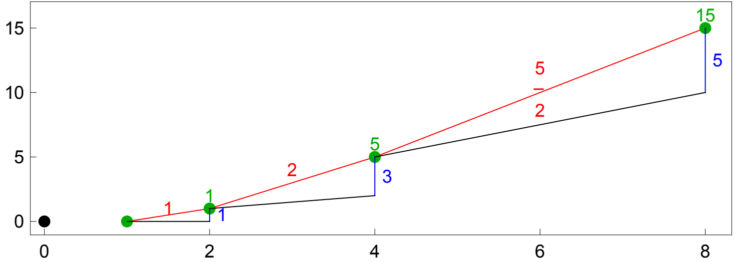
The LMFDB does however always gives the much-harder-to-calculate slope content of the Galois closure L. In our example $G = 9T24$ has $324 = 2^2 3^4$ elements and the slope content of L is

$$\left[\frac{3}{2}, \mathbf{2}, \frac{5}{2}, \mathbf{\frac{17}{6}}\right]_2^2.$$

At this level, the wild slopes are breaks in the Artin upper numbering of the ramification filtration on G. They consist of the wild slopes already **visible** in K, and also some more *hidden* slopes.

2.1. The canonical filtration of a p-adic extension

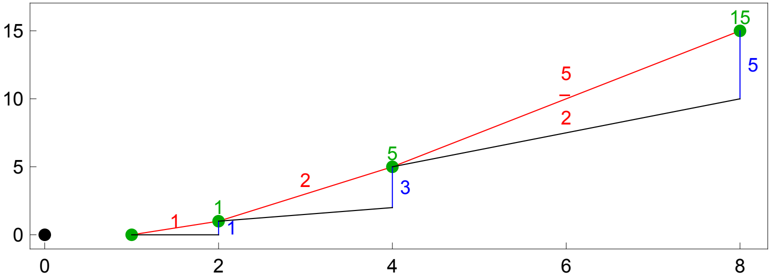
Any $K/F \in F(n)$ has a canonical filtration obtained by climbing from F to K via suitable minimally ramified subextensions. To focus on the main phenomena, we henceforth restrict to $u = t = 1$ so $n = p^w$.



As we'll see, for any 2-adic field F, the picture arises from many octic extensions K/F, e.g. from 32 in the case $F = \mathbb{Q}_2$. The filtration takes the form

$$F = K_0 \subset K_1 \subset K_2 \subset K_3 = K$$

with each $[K_i : K_{i-1}] = 2$.



The numerical invariants are captured in three equivalent ways:

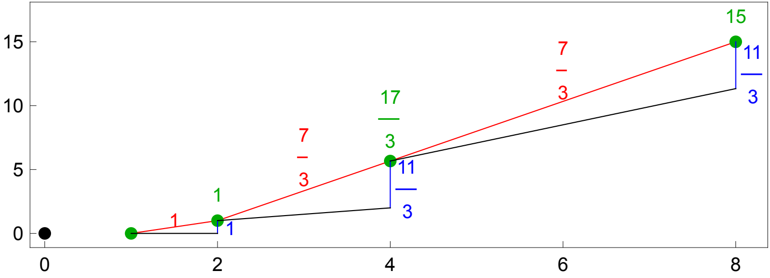
$$\begin{array}{lcll} \textit{heights} & \langle h_1, h_2, h_3\rangle & = & \langle 1, 5, 15\rangle, \\ \textit{slopes} & [s_1, s_2, s_3] & = & [1, 2, \frac{5}{2}], \\ \text{or } \textit{rams} & (r_1, r_2, r_3) & = & (1, 3, 5). \end{array}$$

We switch to focusing on wild ramification only, writing

$$\mathrm{cond}(K_i/K_j) = c(K_i/K_j) - \deg(K_i/K_j) + 1.$$

The definitions are then $h_i = \mathrm{cond}(K_i/K_0)$ and $r_i = \mathrm{cond}(K_i/K_{i-1})$. Accordingly, we also switch from the website's Artin-Fontaine slopes $\hat{s}_i$ to the Serre-Swan slopes s_i via $\hat{s}_i = s_i + 1$.

When the canonical filtration has fewer than w steps we keep a uniform notation that refers to nonexistent fields:



Here the filtration is $F = K_0 \subset K_1 \subset K_2 \subset K_3 = K$. The invariants are

$$I = \langle 1, \frac{17}{3}, 15 \rangle = \left[1, \frac{7}{3}, \frac{7}{3} \right] = \left(1, \frac{11}{3}, \frac{11}{3} \right).$$

When K_i is nonexistent, h_i is no longer forced to be integral. Similarly too, a ram r_j repeated ρ times is now only forced to have a denominator dividing $P_\rho(p) := p^{\rho-1} + p^{\rho-2} + \cdots + p + 1$.

2.2. Conversion formulas

The three ways of describing a ramification invariant I interrelate via

$$h_k \overset{2a}{=} \sum_{j=1}^{k} \phi(p^j) s_j, \qquad h_k \overset{2b}{=} \sum_{j=1}^{k} p^{k-j} r_j,$$

$$s_k \overset{1a}{=} \frac{h_k - h_{k-1}}{\phi(p^k)}, \qquad s_k \overset{3}{=} \frac{r_k}{\phi(p^k)} + \sum_{j=1}^{k-1} \frac{r_j}{p^j},$$

$$r_k \overset{1b}{=} h_k - p h_{k-1}, \quad r_k \overset{4}{=} \phi(p^k) s_k - \phi(p) \sum_{j=1}^{k-1} \phi(p^j) s_j.$$

1a captures the definition of slope as rise/run. 1b emphasizes that rams measure how K_k is more ramified than say the algebra K_{k-1}^p. 2a and 2b are their inversions, each intuitive in their own right. $3 = 1a \circ 2b$ and $4 = 1b \circ 2a$ are less directly intuitive.

3.1. Allowed $r_1 = \cdots = r_\rho$ in one-step extensions

The set of r_1 arising in one-step degree p^ρ extensions of F is very simple and depends only on the ramification index $e = \mathrm{ord}_\Pi(p)$!

In the case of $e = \infty$, i.e. function fields, it is

$$\mathcal{R}_{p,\infty,\rho} = \frac{\mathbb{Z}_{\geq 1} - p\mathbb{Z}_{\geq 1}}{P_\rho(p)} = \frac{\mathbb{Z}_{\geq 1} - p\mathbb{Z}_{\geq 1}}{p^{\rho-1} + p^{\rho-2} + \cdots p + 1}.$$

In the case $e < \infty$, i.e. extensions of $\mathbb{Q}_p$, it is

$$\mathcal{R}_{p,e,\rho} = (\mathcal{R}_{p,\infty,\rho} \cap (0, pe)) \cup \begin{cases} \{pe\} & \text{if } \rho = 1, \\ \{\ \} & \text{if } \rho > 1. \end{cases}$$

The conversion formulas are trivial in the one-step extension context, $s_1 = \frac{r_1}{p-1}$ and $h_\rho = P_\rho(p) r_1$.

3.2. Example: One-step extensions for $p = 2$

From the previous slide for multiplicities $\rho = 1$ and $\rho = 2$,

$$\begin{array}{rcl} \mathcal{R}_{2,\infty,1} & = & \{ \quad 1, \quad \mid \quad 3, \ \ldots \ \}, \\ \mathcal{R}_{2,\infty,2} & = & \{ \ \tfrac{1}{3}, \ 1, \ \tfrac{5}{3}, \mid \tfrac{7}{3}, \ 3, \ \ldots \ \}. \end{array}$$

The cutoff for $e = 1$ is indicated and so the corresponding sets are

$$\begin{array}{rcl} \mathcal{R}_{2,1,1} & = & \{ \quad 1, \quad 2 \ \}, \\ \mathcal{R}_{2,1,2} & = & \{ \ \tfrac{1}{3}, \ 1, \ \tfrac{5}{3} \ \}. \end{array}$$

Over $\mathbb{Q}_2$, the quadratic fields for the rams 1 and 2 are respectively $\mathbb{Q}_2(\sqrt{d})$ for $d \in \{-1, -1*\}$ and $d \in \{2, 2*, -2, -2*\}$, with say $* = 5$. The quartic fields appear on the LMFDB as

$(r_1, r_2) = (1/3, 1/3)$	$(r_1, r_2) = (1, 1)$	$(r_1, r_2) = (5/3, 5/3)$
$x^4 + 2x + 2$ $[4/3, 4/3]_3^2$ S_4	$x^4 + 2x^3 + 2x^2 + 2$ $[2, 2]^3$ A_4	$x^4 + 4x^2 + 4x + 2$ $[8/3, 8/3]_3^2$ S_4
	$x^4 + 2x^3 + 2$ $[2, 2]^2$ D_4	$x^4 + 4x^2 + 2$ $[8/3, 8/3]_3^2$ S_4
	$x^4 + 2x^3 + 6$ $[2, 2]^2$ D_4	

3.3. Occurring invariants I in general extensions

Fix a ground field F with $\mathrm{ord}_\Pi(p) = e$ and consider its totally ramified extensions of degree p^w.

Break up this set of extensions according to their multiplicity vector $m = (m_1, \ldots, m_k)$. Let $M_i = \sum_{j=1}^{i} m_j$. Let $\mathcal{I}_{p,e,m}$ be the set of occurring invariants I. Then necessarily $\mathcal{I}_{p,e,m}$ is in

$$\widehat{\mathcal{I}}_{p,e,m} = \{(\overbrace{r(1), \ldots, r(1)}^{m_1}, \ldots, \overbrace{r(k), \ldots, r(k)}^{m_k}) : r(i) \in \mathcal{R}_{p,ep^{M_{i-1}},m_i}\}.$$

The index gymnastics hide a simple Cartesian product! E.g. $\widehat{\mathcal{I}}_{p,e,(3,2)}$ consists of 5-tuples $(r_1, r_1, r_1, r_4, r_4)$ with $(r_1, r_4) \in \mathcal{R}_{p,e,3} \times \mathcal{R}_{p,p^3e,2}$.

The set $\mathcal{I}_{p,e,m}$ is then the subset of $\widehat{\mathcal{I}}_{p,e,m}$ such that the list of rams $r(1), \ldots, r(k)$, or equivalently the list of slopes $s(1), \ldots, s(k)$, is strictly increasing. The elementary nature of $\mathcal{I}_{p,e,w} = \coprod_m \mathcal{I}_{p,e,m}$ is illustrated by the next slide by $\mathcal{I}_{3,1,2} = \mathcal{I}_{3,1,(1,1)} \coprod \mathcal{I}_{3,1,(2)}$.

3.4. Invariants for tot. ram. nonic 3-adic fields

rams (r_1, r_2) and slopes $[\hat{s}_1, \hat{s}_2]$ with $m = (1,1)$ before $m = (2)$:

$(1,1)$	$(2,1)$	$(3,1)$	$(0.25, 0.25)$
$(1,2)$	$(2,2)$	$(3,2)$	$(0.50, 0.50)$
$(1,4)$	$(2,4)$	$(3,4)$	$(1.00, 1.00)$
$(1,5)$	$(2,5)$	$(3,5)$	$(1.25, 1.25)$
$(1,7)$	$(2,7)$	$(3,7)$	$(1.75, 1.75)$
$(1,8)$	$(2,8)$	$(3,8)$	$(2.00, 2.00)$
$(1,9)$	$(2,9)$	$(3,9)$	
			$(2.50, 2.50)$
			$(2.75, 2.75)$

$[1.5, 1.50]$	$[2, 1.83]$	$[2.5, 2.16]$	$[1.125, 1.125]$
$[1.5, 1.66]$	$[2, 2.00]$	$[2.5, 2.33]$	$[1.250, 1.250]$
$[1.5, 2.00]$	$[2, 2.33]$	$[2.5, 2.66]$	$[1.500, 1.500]$
$[1.5, 1.16]$	$[2, 2.50]$	$[2.5, 2.83]$	$[1.625, 1.625]$
$[1.5, 2.50]$	$[2, 2.83]$	$[2.5, 3.16]$	$[1.875, 1.875]$
$[1.5, 2.66]$	$[2, 3.00]$	$[2.5, 3.33]$	$[2.000, 2.000]$
$[1.5, 2.83]$	$[2, 3.16]$	$[2.5, 3.50]$	
			$[2.250, 2.250]$
			$[2.375, 2.375]$

The Cartesian structure of $\widehat{\mathcal{I}}_{3,1,(1,1)}$ is visible in rams as $\{1,2,3\} \times \{1,2, \;\; 4,5, \;\; 7,8,9\}$, but obscured in slopes.

The Cartesian structure on the $(1,1)$ part is still visible in the total masses to the right, where K/F has mass $|1/\mathrm{Aut}(K/F)|$.

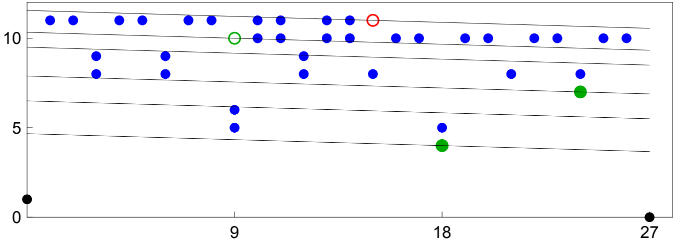
			2
4			2
12	12	18	6
12	12	18	2
36	36	54	6
36	36	54	6
54	54	81	
			6
			6

4.1. From an invariant I to its picture

An invariant $I = h = s = r$ for degree p^w extensions determines a picture in the window $[0, p^w] \times [0, \hat{s}_w]$. For example

$$I = \langle 11, 62, 252 \rangle = [5.5, 8.5, 10.\overline{5}] = (11, 29, 66)$$

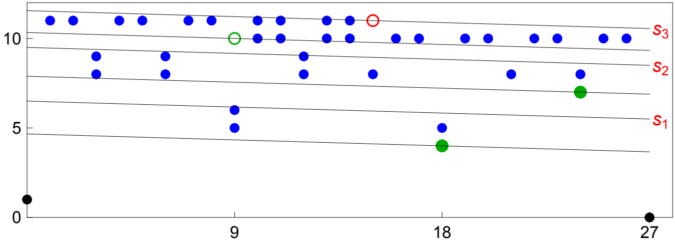
determines



(The points in the u-column will give constraints on the coefficient t_u of the x^u terms in Eisenstein polynomials.)

Q. Can you guess the recipe for passing from I to the picture?

4.2. The recipe for drawing the I-picture, part 1

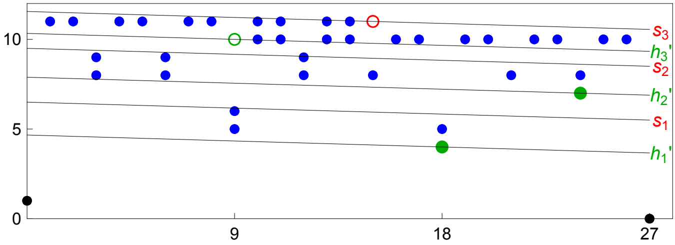


There are w closed bands. The top edge of the i^{th} band B_i goes from $(0, \hat{s}_i)$ to (p^w, s_j). All drawn points (u, v) are integral and, besides $(0, 1)$ and $(p^w, 0)$, occur only in the bands. Write $u' = u/p^w$. Then an integral point $(u, v) \in B_i$ is drawn iff its u' has exact denominator p^i or it's on the boundary. It is drawn solidly iff the first condition holds. There is always a unique point on the lower edge, drawn as ○ or ●. There is at most one point on the upper edge, drawn as ○.

4.3. The recipe for drawing the I-picture, part 2

Define the scaled heights and scaled rams via $h_i' = h_i/p^i$ and $r_i' = r_i/p$, and indicate these variants by double delimiters. So the current example becomes

$$I = \langle\langle 3\tfrac{2}{3}, 6\tfrac{8}{9}, 9\tfrac{1}{3}\rangle\rangle = [5.5, 8.5, 10.\overline{5}] = ((3\tfrac{2}{3}, 9\tfrac{2}{3}, 22)).$$



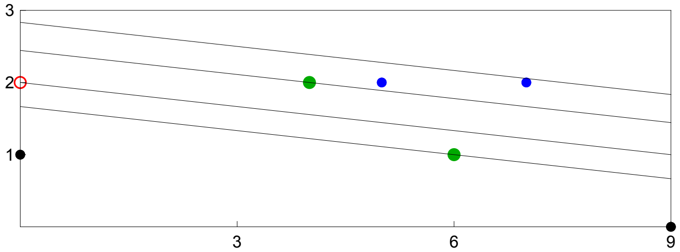
The i^{th} ○ or • is at $(u_i', v_i) = (\langle h_i'\rangle, \lceil h_i' \rceil)$ so that e.g. the first • comes from $3\frac{2}{3}$ and is at $(u_1', v_1) = (\frac{2}{3}, 4)$. Equivalently, the lower edge of B_i goes through (p^w, h_i'). Also, B_i contains exactly $\lfloor r_i' \rfloor$ •'s, and then also a • iff r_i' is nonintegral.

5.1. The Krasner-Monge parametrized polynomial

Index a point (u, v) by the integer $j = p^w(v-1) + u$, so that the i^{th} $\bullet$ or $\circ$ becomes $j = p^w h'_i$. Introduce variables a_j, b_j and c_j for drawn points in bands of the form $\bullet$, $\bullet$, and $\circ$. Form the polynomial

$$\pi + \sum_{(u,v)\text{ as }\bullet} a_j \pi^v x^u + \sum_{(u,v)\text{ as }\bullet} b_j \pi^v x^u + \sum_{(u,v)\text{ as }\circ} c_j \pi^v x^u + x^{p^w}$$

Our earlier example $I = [1, \frac{11}{6}] = ((\frac{2}{3}, 2\frac{1}{3})) = \langle\langle \frac{2}{3}, 1\frac{4}{9} \rangle\rangle$ yields



For $\pi = 3$, it's $(3 + 9c_9) + 9a_{13}x^4 + 9b_{14}x^5 + 3a_6x^6 + 9b_{16}x^7 + x^9$.

Notation for the Krasner-Monge theorem

Let F be a p-adic field with residue field $\mathbb{F}_q$ with $q = p^f$.

For d a divisor of f, the additive map

$$\mathbb{F}_q \to \mathbb{F}_q : k \mapsto k^{p^d} - k$$

has kernel $\mathbb{F}_{p^d}$ and so image $T_d \subset \mathbb{F}_q$ of index p^d.

Choose a uniformizer π and a lift $\kappa \subset \mathcal{O}$ of $\mathbb{F}_{p^f}$. Require $0 \in \kappa$ and write $\kappa^\times = \kappa - \{0\}$. For each divisor d of f, choose a lift $\kappa_d \subset \kappa$ of $\mathbb{F}_q/T_d$, so that $|\kappa_d| = p^d$ and $\kappa_f = \kappa$. For $F = \mathbb{Q}_p$, we always just take $\pi = p$ and $\kappa = \{0, 1, \ldots, p-1\}$.

For a ramification invariant I, let

- α be its number of $\bullet$'s;
- β be its number of $\bullet$'s;.
- $\gamma = \sum_j \gcd(\rho(j), f)$ where j runs over indices of $\circ$'s and $\rho(j)$ it the number of times the corresponding slope is repeated.

Krasner-Monge theorem

Theorem

Let F be a p-adic field with absolute ramification index $e \in \mathbb{Z}_{\geq 1}$ and chosen π and κ_d as on the previous slide. Let $I \in \mathcal{I}_{p,e,w}$ be a possible ramification invariant for degree p^w extensions of F. Consider the polynomials in the corresponding Krasner-Monge family

$$\pi + \sum_{(u,v) \text{ as } \bullet} a_j \pi^v x^u + \sum_{(u,v) \text{ as } \bullet} b_j \pi^v x^u + \sum_{(u,v) \text{ as } \circ} c_j \pi^v x^u + x^{p^w}$$

with $a_j \in \kappa^\times$, $b_j \in \kappa$, and $c_j \in \kappa_{\gcd(\rho(j),f)}$. Then the corresponding extensions are in $F(I)$, with each K represented $\frac{p^\gamma}{|\mathrm{Aut}(K/F)|}$ times.

Corollary

The total number of extensions in $F(I)$ is $\geq (q-1)^\alpha q^\beta$, with equality if $\gamma = 0$.

6.1 The case $I = [\hat{s}_1, \hat{s}_2] = [2, \frac{17}{6}]$ over $\mathbb{Q}_3$

The database says there are 36 fields falling in four packets of nine. As said before, the family is

$$f(a_6, a_{13}, b_{14}, b_{16}, c_9, x) = \\ (3 + 9c_9) + 9a_{13}x^4 + 9b_{14}x^5 + 3a_6x^6 + 9b_{16}x^7 + x^9,$$

Since there is just one c and $f = 1$, the ambiguity parameter is $\gamma = 1$ and each field K has $p^\gamma = 3$ near-canonical defining polynomials. The ambiguity is easily resolved by setting a parameter to 0 and the packets are cleanly described:

$f(1, 2, 0, b_{16}, c_9, x)$	gives $9T13$ and hidden slopes $[5/2]_2$
$f(1, 1, b_{14}, b_{16}, 0, x)$	gives $9T18$ and hidden slopes $[5/2]_2^2$
$f(2, 2, 0, b_{16}, c_9, x)$	gives $9T22$ and hidden slopes $[3/2, 5/2]_2$
$f(2, 1, b_{14}, b_{16}, 0, x)$	gives $9T24$ and hidden slopes $[3/2, 5/2]_2^2$

6.2 The case $I = [\hat{s}_1, \hat{s}_2] = [\frac{5}{2}, \frac{17}{6}]$ over $\mathbb{Q}_3$

The database says that in this case there are 18 fields falling into two packets of nine. The Krasner-Monge family is

$$g(\alpha_{14}, \beta_{12}, \beta_{16}, x) = 3 + 9\beta_{12} + 9\alpha_{14}x^5 + 9\beta_{16}x^7 + x^9$$

Defining polynomials are in this case unique and

$$\begin{array}{ll} g(2, \beta_{12}, \beta_{16}, x) & \text{gives } 9T11 \text{ and hidden slopes } [2]_2 \\ g(1, \beta_{12}, \beta_{16}, x) & \text{gives } 9T18 \text{ and hidden slopes } [2]_2^2 \end{array}$$

In general, resolvent constructions should have nice descriptions via the universal families. For example, $9T13$ from the previous slide and $9T11$ are the same abstract group. The bijection between

- the nine $9T13$ fields defined by $f(1, 2, 0, b_{16}, c_9, x)$ and
- the nine $9T11$ fields defined by $g(2, \beta_{12}, \beta_{16}, x)$

is given by $c_9 = \beta_{12}$ and $b_{16} = \beta_{16} + 1 - \beta_{12}{}^2$.

6.3 The case $I = [\hat{s}_1, \hat{s}_2] = [3/2, \frac{8}{3}]$ over $\mathbb{Q}_3$

The database gives five types of fields. The family is

$$f(a_3, a_{11}, b_{13}, b_{14}, c_{15}) = 3 + 9x^2 a_{11} + 3x^3 a_3 + 9x^4 b_{13} + 9x^5 b_{14} + 9x^6 c_{15} + x^9$$

The five types are

#	μ		
9	3	$f(1, 2, b_{13}, b_{13}+2, c_{15}, x)$	gives $9T12$ and h.s. $[5/2]_2$
18	6	$f(1, 2, b_{13}, b_{13}+\begin{smallmatrix}0\\1\end{smallmatrix}, c_{15}, x)$	gives $9T20$ and h.s $[5/2]_2^3$
9	9	$f(2, 2, b_{13}, b_{14}, \star, x)$	gives $9T18$ and h.s. $[3/2]_2^2$
27	9	$f(2, 1, b_{13}, b_{14}, c_{15}, x)$	gives $9T20$ and h.s $[3/2, 5/2]_2$
9	9	$f(1, 1, b_{13}, b_{14}, \star, x)$	gives $9T24$ and h.d. $[3/2, 2]_2^2$

Here $\star$ can be any element of $\{0, 1, 2\}$ without changing the field. Otherwise, different parameters give different fields.

Commented main references

Much of this material has origin in:

M. Krasner, *Sur la primitivité des corps p-adiques*, Mathematica (Cluj) 13 (1937) 72–191.

Krasner's results were modernized in:

P. Deligne, *Les corps locaux de caractéristique p, limites de corps locaux de caractéristique 0*, in Representations of Reductive Groups over a Local Field (1984), pp. 119–157.

The original database from which the LMFDB database grew:

J. W. Jones and D. P. Roberts, *A database of local fields*, J. Symbolic Comput. 41(1) (2006) 80–97.

A modernization which, like Krasner, emphasizes polynomials:

M. Monge, *A family of Eisenstein polynomials generating totally ramified extensions, identification of extensions and construction of class fields*. Int. J. Number Theory 10 (2014), no. 7, 1699–1727.